



General Certificate of Education
Advanced Level Examination
January 2010

Mathematics

MFP4

Unit Further Pure 4

Monday 25 January 2010 9.00 am to 10.30 am

For this paper you must have:

- a 12-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MFP4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

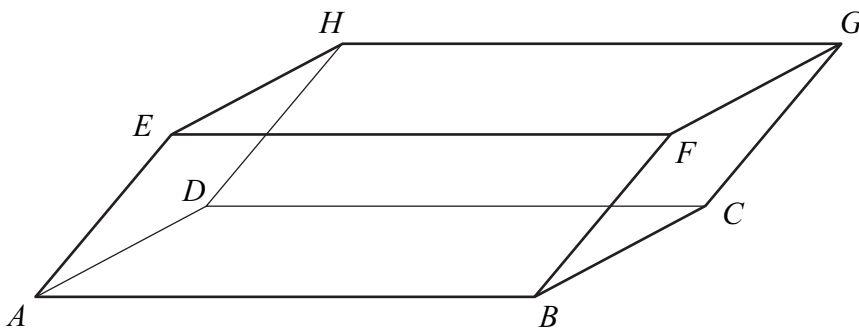
Answer **all** questions.

1 The 2×2 matrix \mathbf{M} represents the plane transformation T . Write down the value of $\det \mathbf{M}$ in each of the following cases:

- (a) T is a rotation;
- (b) T is a reflection;
- (c) T is a shear;
- (d) T is an enlargement with scale factor 3.

(4 marks)

2 The diagram shows the parallelepiped $ABCDEFGH$.



The position vectors of A , B , C , D and E are, respectively,

$$\mathbf{a} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} -3 \\ 10 \\ 4 \end{bmatrix}, \quad \mathbf{d} = \begin{bmatrix} -7 \\ 10 \\ 7 \end{bmatrix} \quad \text{and} \quad \mathbf{e} = \begin{bmatrix} 3 \\ 4 \\ 10 \end{bmatrix}$$

- (a) Show that the area of $ABCD$ is 37. (4 marks)
- (b) Find the volume of $ABCDEFGH$. (2 marks)
- (c) Deduce the distance between the planes $ABCD$ and $EFGH$. (2 marks)

3 The matrices \mathbf{A} and \mathbf{B} are defined in terms of a real parameter t by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & t & 4 \\ 3 & 2 & -1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 15 & -4 & -1 \\ -2t & 4 & 2 \\ 17 & -4 & -3 \end{bmatrix}$$

- (a) Find, in terms of t , the matrix \mathbf{AB} and deduce that there exists a value of t such that \mathbf{AB} is a scalar multiple of the 3×3 identity matrix \mathbf{I} . (5 marks)
- (b) For this value of t , deduce \mathbf{A}^{-1} . (2 marks)

4 (a) Determine the two values of k for which the system of equations

$$\begin{aligned} x - 2y + kz &= 5 \\ (k+1)x + 3y &= k \\ 2x + y + (k-1)z &= 3 \end{aligned}$$

does not have a unique solution. (4 marks)

- (b) Show that this system of equations is consistent for one of these values of k , but is inconsistent for the other.

(You are not required to find any solutions to this system of equations.) (8 marks)

5 The plane transformations T_A and T_B are represented by the matrices \mathbf{A} and \mathbf{B} respectively,

where $\mathbf{A} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 & 5 \\ 5 & 13 \end{bmatrix}$.

- (a) Find the equation of the line which is the image of $y = 2x + 1$ under T_A . (3 marks)
- (b) The rectangle $PQRS$, with area 4.5 cm^2 , is mapped onto the parallelogram $P'Q'R'S'$ under T_B . Determine the area of $P'Q'R'S'$. (2 marks)
- (c) The transformation T_C is the composition

‘ T_B followed by T_A ’

By finding the matrix which represents T_C , give a full geometrical description of T_C . (5 marks)

Turn over for the next question

Turn over ►

- 6 (a) Find the value of p for which the planes with equations

$$\mathbf{r} \cdot \begin{bmatrix} 6 \\ -3 \\ 2 \end{bmatrix} = 42 \quad \text{and} \quad \mathbf{r} \cdot \begin{bmatrix} 4p + 1 \\ p - 2 \\ 1 \end{bmatrix} = -7$$

- (i) are perpendicular; (3 marks)
- (ii) are parallel. (3 marks)
- (b) In the case when $p = 4$:
- (i) write down a cartesian equation for each plane; (2 marks)
- (ii) find, in the form $\mathbf{r} = \mathbf{a} + \lambda \mathbf{d}$, an equation for l , the line of intersection of the planes. (6 marks)
- (c) Determine a vector equation, in the form $\mathbf{r} = \mathbf{u} + \beta \mathbf{v} + \gamma \mathbf{w}$, for the plane which contains l and which passes through the point $(30, 7, 30)$. (2 marks)

7 (a) It is given that $\Delta = \begin{vmatrix} 16 - q & 5 & 7 \\ -12 & -1 - q & -7 \\ 6 & 6 & 10 - q \end{vmatrix}$.

- (i) By using row operations on the first two rows of Δ , show that $(4 - q)$ is a factor of Δ . (2 marks)
- (ii) Express Δ as the product of three linear factors. (4 marks)
- (b) It is given that $\mathbf{M} = \begin{bmatrix} 16 & 5 & 7 \\ -12 & -1 & -7 \\ 6 & 6 & 10 \end{bmatrix}$.
- (i) Verify that $\begin{bmatrix} 2 \\ 5 \\ -7 \end{bmatrix}$ is an eigenvector of \mathbf{M} and state its corresponding eigenvalue. (3 marks)
- (ii) For each of the other two eigenvalues of \mathbf{M} , find a corresponding eigenvector. (7 marks)
- (c) The transformation T has matrix \mathbf{M} . Write down cartesian equations for any one of the invariant lines of T . (2 marks)

END OF QUESTIONS